running back. It is found that the initial condition is one in which moon and earth rotate, as though fixed together, in 5 hrs. 40 mins.; and that this condition is one of dynamical instability, so that the moon must either have fallen into the earth, or have receded from it, and have then gone through the changes which were traced backwards.

From this and other considerations it is concluded that, if the moon and earth were ever molten viscous masses, then it is highly probable that they once formed parts of a common mass.

The rest of the paper is occupied with a number of miscellaneous propositions, and with a discussion of the physical significance of the results obtained.

I will here only mention that the case of the Martian satellites appears to me a very striking corroboration of the applicability of these views to the solar system, whilst the Uranian system of satellites is, at first sight, unfavourable.

A whole series of problems, some of them of great difficulty, still await solution; and not until they are solved will it be possible either decisively to accept or reject the modified form of the nebular hypothesis, to which my results obviously point.

(Postscript.) Added November 8th, 1878.

A subsequent investigation has shown that, although the amount of heat which might be generated by internal friction in the earth might be very great, yet its distribution would be such that it could scarcely sensibly affect Sir W. Thomson's investigation of the secular cooling of the earth.

III. "Problems connected with the Tides of a Viscous Spheroid." By G. H. Darwin, M.A., Trinity College, Cambridge. Communicated by J. W. L. Glaisher, F.R.S. Received November 14, 1878.

(Abstract.)

In this paper certain problems are treated, which were alluded to in two previous papers on the Tides and Precession of a viscous spheroid.* For brevity the spheroid is spoken of as the earth, and the disturbing body as the moon.

I. *Secular Distortion of the Spheroid, and certain Tides of the second order.*

The distortion arises from the unequal distribution of the tidal frictional couple over the surface of the spheroid.

* Read before the Royal Society on May 23 and December 19 respectively.

In forming the theory of tides, it was assumed that the action of the tidal protuberance on any element of the surface of the mean sphere was entirely normal to the sphere, and consisted of the weight of the prismatic element of the tidal protuberance, which stands on the element of surface. This is not rigorously correct, because, if it were so, there would be no couples tending to alter the diurnal rotation and obliquity of the earth. The effects of these couples were considered in the paper on "Precession," but the tidal protuberance was there assumed to be instantaneously rigidly connected with the mean sphere. The present problem is concerned with the non-rigid attachment of the protuberance to the sphere.

A sphere is supposed to be distorted into any form differing infinitesimally from the true sphere, and to be acted on by any external disturbing potential. It is then found what tangential stress must be supposed to act across the base of any prismatic element of the protuberance, in order that the equilibrium of that element may be maintained, the pressures transmitted by the four contiguous elements being taken into account. It appears that if the protuberance has the equilibrium form, due to the external disturbing potential, then there is no tangential stress between the true sphere and the protuberance. But since the tides of a viscous spheroid lag, the form of the viscous tidal protuberance is not one of equilibrium, and there is such a tangential stress across the base of each element of the protuberance. It is obvious that these tangential stresses may produce a continued distortion of the spheroid.

The problem, as applicable to the earth, is treated in the simple case where the obliquity to the ecliptic is zero, and where there is only one disturbing body or moon.

The sum of the moments of the tangential stresses about the axis of rotation gives the tidal frictional couple, and its form is found to agree with that found by a different method in the paper on "Precession."

When the earth's rotation is taken into account, it appears that the component along the meridian of tangential stress at any point of the surface is periodic in time; whilst one part of the component perpendicular to the meridian is periodic, and the other non-periodic. The periodic parts of the component tangential stresses give rise to small tides of the second order (varying as the square of the tide-generating force), and are neglected, but the non-periodic part gives rise to a secular distortion.

Since the earth's rotation as a whole is retarded, therefore the distorting tangential stresses all over the surface constitute a non-equilibrating system of forces, and in order to find the distortion of the globe, they must be deemed to be equilibrated by the effective forces due to the inertia of the slackening diurnal rotation. These

effective forces give bodily forces in the interior, the sum of whose moments about the axis of rotation is equal and opposite to the tidal frictional couple. The problem is thus reduced to finding the distortion of a sphere subject to bodily force equilibrated by surface action, and it is solved by Sir W. Thomson's method of finding the internal strain of an elastic sphere under like conditions, although here the bodily force has no corresponding potential function.

The solution shows that the distortion consists in a simple cylindrical motion round the axis of rotation, each point moving from east to west with a linear velocity proportional to the cube of its distance from that axis.

The distortion of the surface of the globe consists of a motion in longitude from west to east, relatively to a point in the equator, the rate of change of longitude being proportional to the square of the sine of the latitude.

Numerical calculation shows, however, that in the later stages of the earth's history (the development being supposed to follow the laws found in the paper on "Precession"), the distortion must have been very small. With a certain assumed viscosity, it is found that, looking back 45,000,000 years, a point in latitude 60° would lie 14′ further east than at present. From this it follows, that this cause can have had little or nothing to do with the crumpling of geological strata.

As, however, the distorting force varies inversely as the sixth power of the moon's distance, it seems possible that in the very earliest stages this cause may have had sensible effects. It is, therefore, noteworthy that the wrinkles raised on the surface would run north and south in the equatorial regions, with a tendency towards north-east and south-west in the northern hemisphere, and north-west and south-east in the southern one. The intensity of the distorting force at the surface varies as the square of the cosine of the latitude.

An inspection of a map of the earth shows that the continents (or large wrinkles) conform more or less to this law. But Professor Schiapparelli's map of Mars* is more striking than that of the earth, when viewed by the light of this theory; but there are some objections to its application to the case of Mars. If, however, there is any truth in this, then it must be postulated, that after the wrinkles were formed the crust attained sufficient local rigidity to resist the obliteration of the wrinkles, whilst the mean figure of the earth adjusted itself to the ellipticity appropriate to the slackening diurnal rotation; also, it must be supposed that the general direction of the existing continents has lasted through geological history.

The second question, considered in the first part, deals with the

* "Memorie della Società degli Spettroscopisti Italiani," 1878, vol. vii.

non-rigid attachment of the permanent equatorial protuberance to the mean sphere. It is shown that the precessional and nutational couples will give rise to certain tides of the second order (varying as the tide-generating force multiplied by the precessional constant), but not to any secular shifting of the surface over the interior, as has been supposed would be the case by some writers.

II. *Distribution of Heat generated by Internal Friction, and the secular cooling of the Spheroid.*

In the paper on "Precession" it was shown by the theory of energy, that a very large amount of heat might have been generated inside the earth by friction, but the investigation gave no indication as to its distribution. The problem is here considered by finding the amount of work done per unit of time on each element of the interior in the course of the tidal distortion.

The aggregate work done on the whole globe is found to be the same as that given by simple considerations of energy. The rate of work is equal to the tidal frictional couple multiplied by the relative angular velocity of the moon and earth; but this simple law arises out of a complex law of internal distribution. By far the larger part of the work done, or heat generated, is found to be in the central portion.

My first impression was that the large amount of heat, which might be generated, would serve to explain in part the observed increase of underground temperature; but the solution of a certain problem concerning the cooling of an infinite slab of rock 8,000 miles thick, in which heat is being generated according to a certain law of distribution, shows that the frictional heat could not possibly explain a rate of increase of underground temperature near the earth's surface of more than 1° F. in 2,600 feet.

It follows, therefore, that Sir W. Thomson's investigation of the secular cooling of the earth cannot be sensibly affected by this cause.

III. *The Effects of Inertia in the Forced Oscillations of Viscous, Fluid, and Elastic Spheres.*

In the theory of tides used hitherto the effects of inertia have been neglected. It was, however, shown that this defect in the theory could not have an important influence, unless the frequency of the tides was much greater than that of those generated by the moon at the present time. Nevertheless it was desirable to determine what the effect of inertia actually is.

This part of the present paper contains a second approximation to the theory of tides of a viscous spheroid.

The first approximation, being that given in the paper on "Tides," is here used to give a value to the terms introduced in the equations of motion by inertia. Physically the terms so introduced are equivalent to an addition to the bodily force which tends to produce the tidal distortion. The problem is then treated by a process parallel to that used by Sir W. Thomson in his statical problem concerning the strain of an elastic sphere. The analytical investigation is long and complicated, and it will here suffice to state the result with regard to the form of the tidal protuberance, when the tide-generating potential is of the second order of harmonics. It is as follows:—If a be the radius, w the density, g mean gravity, and $\mathfrak{g}=\frac{2g}{5a}$, v the "speed" of the tide, η the alteration of phase; so that $\eta \div \mathrm{v}$ is the "lag," and υ the coefficient of viscosity.

Then $$\eta-\frac{79\mathrm{v}^2}{150\mathfrak{g}}\sin\eta\cos\eta=\text{arc-tan}\,\frac{19\upsilon\mathrm{v}}{5\mathfrak{g}wa^2}$$

And the height of tide is equal to the equilibrium tide of a perfectly fluid spheroid multiplied by—

$$\cos\eta\left(1+\frac{79\mathrm{v}^2}{150\mathfrak{g}}\right)$$

This shows that the defect of the first approximation was such that for a given speed, the lag is a little greater, and for a given lag, the height of tide is a little greater than was supposed.

It is then shown that this correction to the theory of tides will scarcely make any appreciable difference in the results of the integration, by which the secular changes in the configuration of the earth and moon, were found in the paper on "Precession;" and especially that it makes *no* difference as to the critical relationship between the month and day, for which the rate of change of obliquity vanishes. The most important influence of the new theory is on the time, and it appears that the time occupied by the changes, above referred to, is overstated by perhaps $\frac{1}{40}$th part.

A comparison is then made of the preceding theory with that of the forced vibrations of a fluid sphere. This shows that when η is zero (*i.e.*, when viscosity graduates into fluidity), the $\frac{79}{150}$ which occurs in the above expressions should properly be $\frac{1}{2}$ or $\frac{75}{150}$. The discrepancy between the 79 and 75 is explained by the fact that in approaching the problem of fluidity from the side of viscosity, we suppose in the first approximation, that the motion of the interior of the sphere is vortical, whereas in reality it is not so.

In conclusion, it is proved that analysis, of almost identically the same character as that for the problem of the viscous sphere, is applicable to the case of an incompressible elastic sphere, and that inertia has the effect of increasing the ellipticity of the tidal spheroid, as given

by Sir W. Thomson's statical theory, in the proportion of $1+\frac{79v^2}{150(r+g)}$ to unity, where v is the speed of the tide, and r is the quantity defined in Thomson and Tait's Nat. Phil., § 840 (28), viz., $\frac{19}{5wa^2}\times$ the coefficient of rigidity.

The last part of the paper contains a discussion of results, and a non-mathematical summary of what precedes.

IV. "On the Influence of Light upon Protoplasm." By ARTHUR DOWNES, M.D., and THOMAS P. BLUNT, M.A. Oxon. Communicated by J. MARSHALL, F.R.S., Surgeon to University College Hospital. Received October 9, 1878.

This paper is in continuation of, and supplementary to, a previous communication* in which we recorded the first part of an investigation on the effect of light upon *Bacteria* and other organisms associated with putrefaction and decay. The chief conclusions to which those observations led us were briefly as follow:—

(1.) Light is inimical to, and under favourable conditions may wholly prevent, the development of these organisms; its action on *Bacteria* being more energetic than upon the mycelial (and torulaceous) fungi which are prone to appear in cultivation-fluids.

(2.) The fitness of the cultivation-fluid as a nidus is not impaired by insolation.

We found also that tubes, containing a cultivation-fluid and plugged with cotton-wool, when removed to a dark place after exposure to the sun for a sufficient period, remained perfectly clear and free from organisms for months, and we naturally thought that the contents had been reduced to permanent sterility. The following facts, however, compel us to suspend for the present our conclusions on this point. Of the many tubes which we insolated last year we finally kept only three. Two of these—containing Pasteur solution of the composition given in our former paper—had been exposed to sunlight for three weeks in June, 1877; the third tube contained urine and had been insolated for about two months—commencing July 26th. In each case corresponding tubes which were covered with laminated lead, so as to exclude light, had swarmed with *Bacteria* in the course of two or three days, but the three tubes of which we speak not only were perfectly pellucid at the time they were removed from the light but, although kept in a warm room, remained clear all through the winter. On February 25th, 1878, however,—eight months after we had placed

* "Proc. Roy. Soc.," vol. xxvi, p. 488.